Gravitational potential is a function of the material within r; pressure is a function of the material inside and outside of r. The material density at any internal radius is a product of both, where pressure sets the baseline density. For the sake of simplicity, we will assume that baseline density varies linearly with r, peaking to $\rho_{\max}$ at ($r = 0$):

$$\rho_{bn}(r) = \rho_{\max} - \left(\frac{\rho_{\max} - \rho_{n_0}}{R_{sr}}\right) r \tag{13.27}$$

where R_{sr} is the radius of a compact object's surface. Since pressure is a function of exterior material, an object's nuclear baseline density returns to ρ_{n_0} at its surface:

$$\rho_{bn}(R_{sr}) = \rho_{n_0} \tag{13.28}$$

The mass at any radius R is a function of the density profile for ($r < R$), so rewrite Equation (13.26) to define the density at a radius R in terms of the total mass of the adiabatic density profile below it:

$$\rho(R) = \left(\sqrt{1 - \frac{2G\left(\int_{r=0}^{R} \rho(r) dV\right)}{c^2 R}}\right)^3 \rho_{bn}(R) \tag{13.29}$$

This nonlinear equation has boundary conditions of:

$$\int_{r=0}^{R_{sr}} \rho(r) dV = M \tag{13.30}$$

$$\rho(r > R_{sr}) = 0 \tag{13.31}$$

$$\rho(r = 0) = \rho_{\max} \tag{13.32}$$

where M is the mass of the compact object. For any given object, there is a density profile $\rho(r)$ that is consistent with the density at each radius R as defined by Equation (13.29), and which integrates to a total mass M over the volume defined by the radius of the object's surface, R_{sr}.

A galaxy's central black hole will be referred to as its *core*. The mass of the Milky Way's core is about three million times that of our sun.⟨1.9⟩ Its interior density profile is shown below:

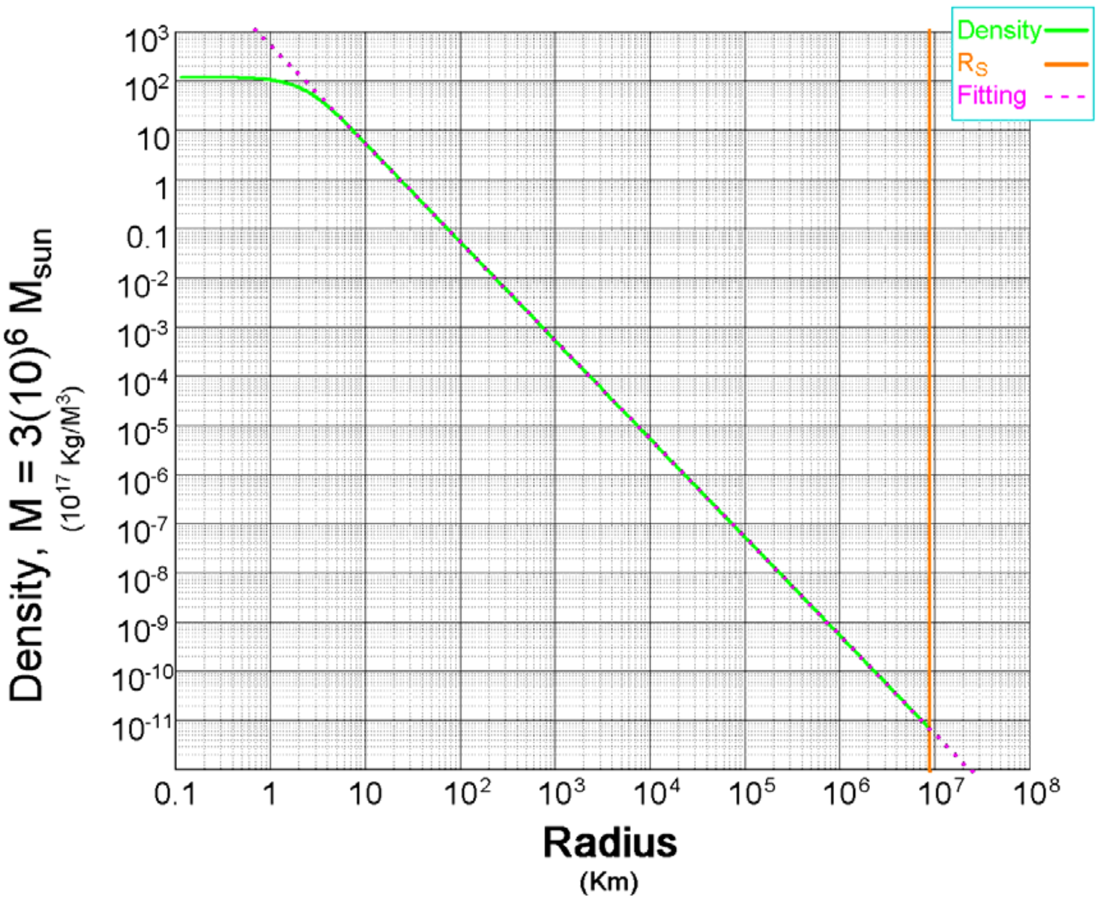

Figure (13.7) Density distribution of the black hole at the center of the Milky Way

It is assumed that a black hole this heavy can compress its innermost matter to hyperdensity ($1.2(10)^{19}$ kg/m^3). However, although the material at the center of the Milky Way's core is in all likelihood hyperdense, its peak interior density has no effect on the density profile in its outer layers or at its surface. The Milky Way core's Schwarzschild radius is shown as a orange vertical line near 10^7 km. Its material surface extends slightly beyond this, but the deviation is too small to appear in Figure (13.7). Most significantly, *the ratio between the total mass and volume of Equation (13.29)'s distribution is within 6 parts per billion (ppb) of a Schwarzschild mass/volume ratio for the galactic core*. Even though matter is not infinitely compressible, when black holes are large enough for their surfaces to decouple from the hyperdense material deep within their interior, they approach a Schwarzschild relationship. The source code for this calculation can be found at *www:nullphysics.com*.

Our calculations were simplified by reducing baseline density linearly with radius, but Figure (13.7)'s density profile emerges *for virtually any baseline density function*. So although